Mathematical Modeling and Experimental Analysis for Flow of Emulsions in Porous Media

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Abstract

It has been suggested that oil migrates through reservoir sands in the form of a fine, emulsions of oil-in-water, and that oil accumulations occur where the stream enters finer-grained rock such as silt or shale. Since emulsion is a non-Newtonian fluid and a power law fluid, the pressure drops of the emulsion (oil-in-water) depend on the viscosity, density, and other parameters such as consistency factor, K and power law index, n. Darcy's equation, Rate equation and Equation of State were combined and used to develop the mathematical model. Experimental analyses were also carried out to validate the mathematical model. The average percentage error value between the calculated results and the experimental results was 6.57 %. Plots of correlation factors (coefficient differences) were also plotted to show and describe the disparities between the obtained model and other models. The significances of these plots were evident as it explained the range of close proximities of other models to the model. It was concluded that for pseudo-plastic type of emulsions, the pressure drop in the porous medium is inversely proportional to the viscosity of the emulsion.

Keywords

Oil Migrates; Reservoir Sands; Emulsions; Non-Newtonian Fluid

Introduction

Crude oil is seldom produced alone. It is generally commingled with water, which creates a number of problems during oil production. Produced water occurs in two ways: some of the water may be produced as free water (i.e., water that will settle out fairly rapidly), and some of the water may be produced in the form of emulsions. A crude-oil emulsion is a dispersion of water droplets in oil. Produced oilfield emulsions can be classified into three broad groups:

- Water-in-Oil (W/O) Emulsions
- Oil-in-Water (O/W) Emulsions
- Multiple or Complex Emulsion.

The three basic conditions necessary for the formation of an emulsion are;

The two liquids that form the emulsion must be immiscible.

There must be sufficient agitation to disperse one liquid as droplets in the other. There must be an emulsifying agent present. There are also a host of other physical parameters which are considered in the formation of emulsion, emulsion type and its stability.

These include: Quality of the emulsion, interfacial tension effects, temperature, pressure, turbulence, and viscosity of external and external phases, shear rate of the emulsion, suspended particle concentration and size distribution.

What are Emulsions?

Oil spilled at sea can form water-in-oil emulsions, consisting of water droplets suspended in an oil matrix. Once oil

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enters the sea it moves on the water's surface by advection and spreading. This movement increases the exposure area of the oil to subsequent "weathering processes," one of which is emulsification, [1, 12]. Although the process of emulsification is not well understood, it is known that some type of mixing energy, such as breaking waves, is needed to form an emulsion. In addition, it is speculated that the rate of emulsification is correlated with wind speed, a major source of mixing energy. Some emulsions have high water content and are relatively stable, maintaining their chemical attributes for several days; these are sometimes referred to as "chocolate mousse." With the uptake of water, the properties of the oil can change dramatically. The viscosity can increase by orders of magnitude and the density can increase to nearly that of water. In fact, many skimmers or other control equipment are ineffective once oil becomes emulsified.

One classification scheme separates emulsions according to stability and includes specific operational definitions for stable, unstable, and meso-stable emulsions.

From a purely thermodynamic point of view, an emulsion is an unstable system. This is because there is a natural tendency for a liquid/liquid system to separate and reduce its interfacial area and hence, its interfacial energy. However, most emulsions are stable over a period of time (i.e., they possess a kinetic stability). Produced oilfield emulsions are classified on the basis of their degree of kinetic stability as follows: Loose emulsions, medium emulsions, tight emulsions.

Emulsions are considered special liquid-in-liquid colloidal dispersions, Toth et-al; 2001, Buckley and Leverett, 1941, [14, 2] reported that kinetic stability is a consequence of a small droplet size and the presence of an interfacial film around the water droplets. Emulsion kinetic stability is attained by stabilizing agents (or emulsifiers) that could be naturally occurring in the crude oil (asphaltenes, for example). These stabilizers suppress the mechanisms involved (i.e., sedimentation, aggregation or flocculation, coalescence, and phase inversion) in emulsion breakdown.

The objectives of this study are:

To model equation for pressure drop during flow of emulsions in porous media.

To carry out laboratory experiments for flow of emulsions in porous media.

To compare and validate the experimental results (data) and other models obtained from the literature withmodel prediction (new model).

Types of Fluids

Fluids may be classified in two ways: either its response to effect of pressure or shear stress. If volume is independent of P, T, then fluid is incompressible. If volume is dependent on P, T, then fluid is compressible. In gases and most pure liquids, the ratio of shear stress to shear rate is constant, hence they are called Newtonian. In some liquids especially those containing the second phase in suspension, the ratio is not constant and apparent viscosity of the fluid is a function of the rate of shear, then fluid is said to be non-Newtonian. These non-Newtonian fluids exhibit rheological properties.

Newton's law of viscosity can be expressed as:

$$\tau \, \alpha \, \delta$$

The above equation expresses shear stress as being proportional to shear rate.

$$\tau \, \alpha - \frac{dv}{dz}$$

$$\tau = \kappa\delta$$

$$\tau = \mu\delta$$

Where τ = shear stress, δ = shear rate,

μ = viscosity, and κ = consistency factor

For many fluids however, τ is not proportional to δ, [3, 11, 14]

Non-Newtonian fluids can be divided into:

a) Pseudo-plastic (shear thinning): The apparent viscosity is not constant, it decreases with increase in the rate of shear. Examples are polymer melt, paper pulp, and polymer solutions.

b) Dilatants (shear thickening): This occurs when apparent viscosity increases with shear rate. Examples are starch suspension, potassium nitrate, gum arabic solutions.

c) Bingham plastic: Examples are rock suspensions, clay solutions.

Pseudo-plastic, Dilatants and Bingham plastic materials are time independent non-Newtonian fluids, i.e. apparent viscosity depends only on the rate of shear at any particular moment and not on the time to which shear rate is applied. Some non-Newtonian fluids however are time dependent but they are rare in practice.

Generally, attempts have been made to formulate mathematical models to represent the rheological behavior of non-Newtonian. The simplest relationship is the power law equation;

$\tau = \kappa\delta^n$

κ = consistency coefficient

n = Power law index

For pseudo-plastic, $n < 1$

For Dilatants, $n > 1$

For Newtonian, $n = 1$

From $\tau = \kappa\delta^n$

$$\mu_a = \frac{\tau}{\delta}$$

$$\mu_a = \kappa\delta^{n-1}$$

The Mathematical Model

Consider the steady laminar flow of a fluid of constant density, ℓ, in a tube of length L and radius r. We assume that there are no end effects; however, this assumption could be removed by using appropriate corrections.

The momentum flux or shear stress distribution in a circular section perpendicular to the tube axis is given as

$$\tau = \frac{\Delta p}{2L} r \tag{1}$$

Using the equation (1) above with the theoretical analysis given by Alvarado and the studies by [4, 6, 8, 13] and [5, 7, 9-10]. The basic equations are

$$\nabla = \frac{q}{\prod r^2} \tag{2}$$

$$\gamma_r = \left(-\frac{dV}{dr}\right) = \frac{3n+1}{4n}\left(\frac{8V}{d}\right) \tag{3}$$

$$\gamma_r = \frac{r\Delta p}{2L} \tag{4}$$

$$\gamma_r = K\left(\frac{8\nabla}{d}\right)^n \tag{5}$$

The flow behavior index, n, is given by

$$n = \frac{d \ln\left(\frac{d\Delta p}{4L}\right)}{d \ln\left(\frac{8V}{d}\right)} \tag{6}$$

The value $8V/d$ can be considered as an apparent shear rate. Thus,

$$\gamma_a = \frac{8V}{d} \tag{7}$$

For many fluids K *and* n are constant over wide ranges of γ_a *or* τ_r.

By definition, the apparent viscosity, μ_a, is given by

$$\mu_a = \frac{\tau_r}{\gamma_r} \tag{8}$$

Using equation (3) through (5) to rewrite equation (8) in terms of the rheological constants K *and* n

$$\mu_a = \left(\frac{4n}{3n+1}\right) K \left(\frac{8\nabla}{d}\right)^{n-1} \tag{9}$$

From Darcy's law

$$q = -\frac{KA}{\mu}\left(\frac{dp}{dl}\right) \tag{10}$$

This equation is valid when flow is laminar.

When Reynold's number $\leq$ 0.1, then,

$$-\frac{dp}{dl} = av\,; \quad a = \frac{\mu}{K}\,; \quad v = \frac{q}{\mu}$$

If however, the flow velocity is high, it is necessary to correct for kinetic energy effect.

Forcheimer proposed a correction factor for the kinetic energy effect.

The Forcheimer equation is written as

$$-\frac{dp}{dl} = av + bv^2 \tag{11}$$

$$a = \frac{\mu}{K}$$

$$b = \beta\ell \quad = correction\ \ factor$$

$$\beta = turbulence\ correction\ \ factor$$

$$\ell = density$$

From equation (2)

$$-\frac{dp}{dl} = av\left(1 + \frac{b}{a}v\right) \tag{12}$$

Since $\frac{b}{a} = \frac{\beta\ell}{\mu}K$

$$= av\left(1 + \frac{K\beta\ell}{\mu}v\right)$$

$$-\frac{dp}{dl} = a\delta v$$

Where $\delta = \left(1 + \frac{K\beta\ell}{\mu}v\right)$

$$q = -\left(\frac{KA}{\mu\delta}\right)\left(\frac{dp}{dl}\right) \tag{13}$$

Similarly, permeability K for basic linear equation for horizontal flow can be calculated.

From $q = -\frac{KA}{\mu}\left(\frac{dp}{dl}\right)$

$$\frac{q\mu}{KA}\int_{l_1}^{l_2} dl = -\int_{p_1}^{p_2} dp$$

$$\frac{q\mu}{KA}(l_2 - l_1) = (p_1 - p_2)$$

$$q = \frac{KA(p_1 - p_2)}{\mu(l_2 - l_1)}$$

$$= \frac{KA}{\mu L}(p_1 - p_2)$$

Thus, $K = \frac{q\mu L}{A(p_1 - p_2)}$ (14)

Introducing flow potential into Darcy's law,

For linear system, we have

$$q = \frac{KA}{\mu v}\left(\frac{d\Phi}{dS}\right)$$

$$= \frac{KA(\Phi_1 - \Phi_2)}{\mu v \Delta L} \tag{15}$$

$\Phi =$ *flow potential*

$S \equiv x, y \,\&\, z$

For radial flow;

$$q = \frac{2\Pi Kh(p - p_w)}{\mu \ln\left(r/r_w\right)} \tag{16}$$

$$p = p_w + \frac{q\mu}{2\Pi Kh}\ln\left(r/r_w\right) \tag{17}$$

Introducing flow potential,

$$q = \frac{2\Pi Kh(\Phi - \Phi_w)}{\mu v \ln\left(r/r_w\right)} \tag{18}$$

From equation (17) above of radial flow equation, Muskat equation is given by

$$p = A + B \ln r$$
$$where\ A\ is\ a\ constant \tag{19}$$
$$B = \frac{q\mu}{2\Pi Kh}$$

$$p = A + \frac{q\mu}{2\Pi Kh} \ln r \tag{20}$$

$$p(r) = A + \frac{q\mu}{2\Pi Kh} \ln r \tag{21}$$

Equation (21) is Muskat equation for radial flow.

$$P = P_i + \left[\frac{q}{2\pi h}\right]^n \frac{K}{\mu} \frac{r^{n-1}}{n+1} E_i(-\beta) \tag{22}$$

This is the model equation that captures the pressure drop for flow of emulsions in porous media.

The governing equations for these are:

i). Conservation of mass equation

ii). Equation of state

iii). Rate equation (Darcy's) law

These equations are combined together to obtain the diffusivity equation for a non-Newtonian fluid, and subsequently solved to obtain the model.

The Experimental Analysis

The materials used are tube of transparent plastic cylinder, emulsion container, sand pack (fine-grained material), measuring cylinder, stopwatch.

Experimental Procedure

The laboratory experiment was carried out with the use of kerosene as the disperse phase, and water as the continuous phase. The emulsion was generated by mixing the two components with the use of a mixer. The emulsion was agitated rigorously for 20 mins so as to generate homogenous mixture. 500 ml volume of water and 50 ml volume of kerosene were used throughout the experiment. The quantity of grounded clay was varied in each of the experiment. Before mixing, some amount of grounded clay was added to the mixture. This served as the emulsifying agent. After mixing, the viscosity of the emulsion was measured. The readings of viscosity were taken at 600 rpm and 300 rpm. Since the experiment is a one-phase flow, this emulsion was left for 4 hrs before use so as to be sure if it does not separate into two phase, after which it was now ready for the experiment to be carried out.

TABLE I. PRESSURE DROPS AND VISCOSITIES FOR MODEL PREDICTION AND EXPERIMENTAL

Quantity of Grounded Clay (g)	**Time Range (Secs)**	**Viscosity (cp)**	**Pressure (psi) (Model Prediction)**	***Pressure (psi) (Experimental)***
25	(300 – 1500)	0.10	4.33	4.62
40	(300 – 1500)	0.15	2.74	2.92
55	(300 – 1500)	0.25	1.56	1.66
70	(300 – 1500)	0.35	1.05	1.12
85	(300 – 1500)	0.45	0.77	0.82

TABLE II. COMPARISON OF EXPERIMENTAL, MODEL AND OTHER PREDICTED MODELS OF CIVAN, MARADEN AND MCAULIFFE

Quantity of Grounded Clay (g)	Time Range (Secs)	Viscosity (cp)	Pressure (psi) Experimental	Pressure (psi) Model	Pressure (psi) Civan Model	Pressure (psi) Marsden Model	Pressure (psi) McAuliffe Model
25	(300 – 1500)	0.10	4.62	4.33	6.01	3.98	5.32
40	(300 – 1500)	0.15	2.92	2.74	4.44	2.57	3.82
55	(300 – 1500)	0.25	1.66	1.56	3.01	1.98	2.61
70	(300 – 1500)	0.35	1.12	1.05	2.23	1.56	1.78
85	(300 – 1500)	0.45	0.82	0.77	1.62	1.13	1.09

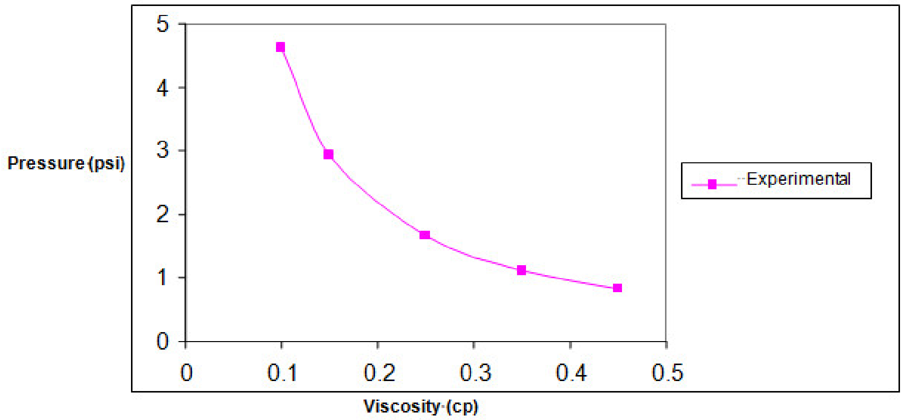

FIGURE 1. PRESSURE DROPS AGAINST VISCOSITY FOR EXPERIMENTAL DATA

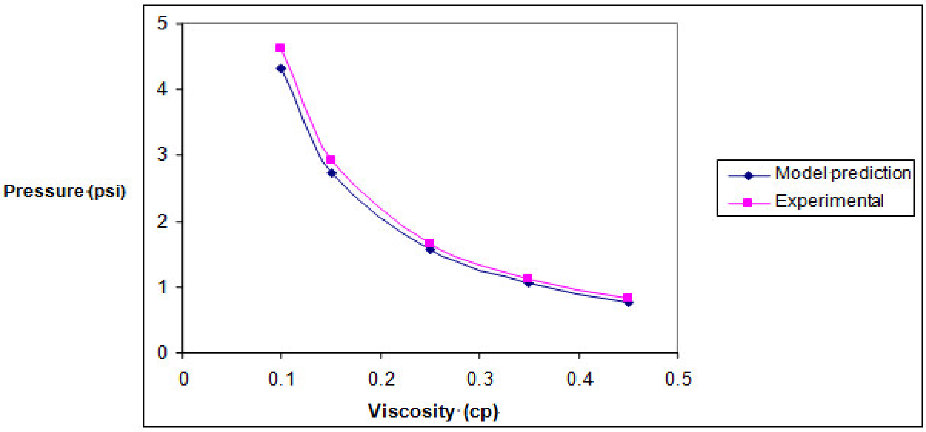

FIGURE 2. PRESSURE DROPS AGAINST VISCOSITY FOR BOTH EXPERIMENTAL AND MODEL

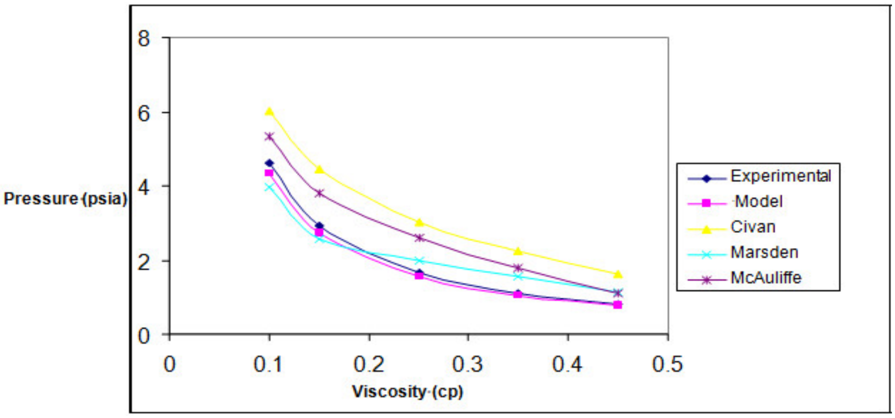

FIGURE 3. COMPARISON OF PRESSURE DROPS AGAINST VISCOSITIES EXPERIMENTAL,MODEL, CIVAN, MARSDEN AND McAuliffe

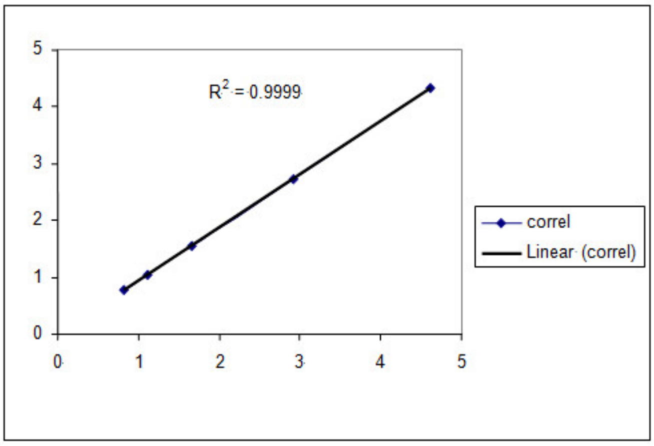

FIGURE 4. CORRELATION BETWEEN THE EXPERIMENTAL AND MODEL

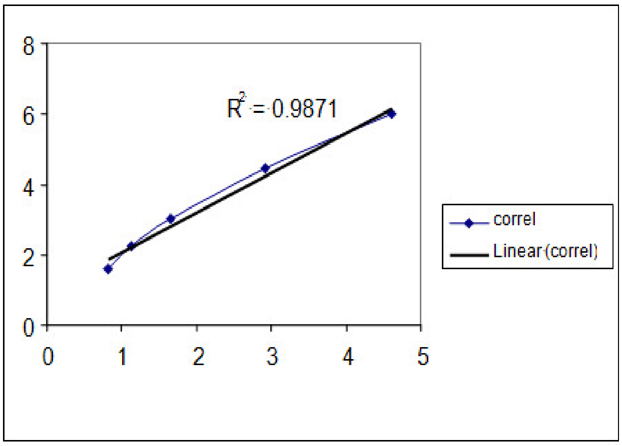

FIGURE 5. CORRELATION BETWEEN CIVAN AND MODEL

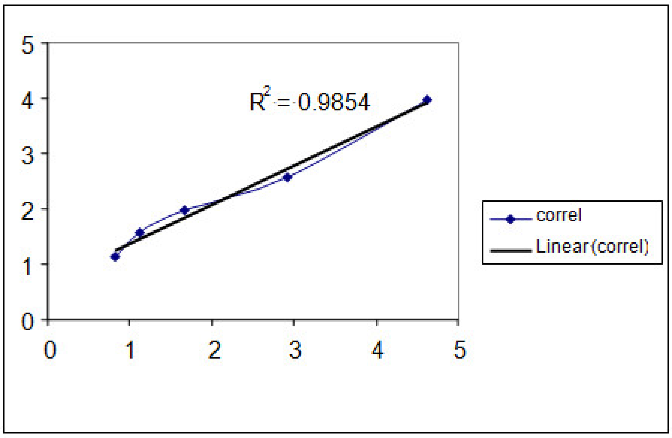

FIGURE 6. CORRELATION BETWEEN MARSDEN AND MODEL

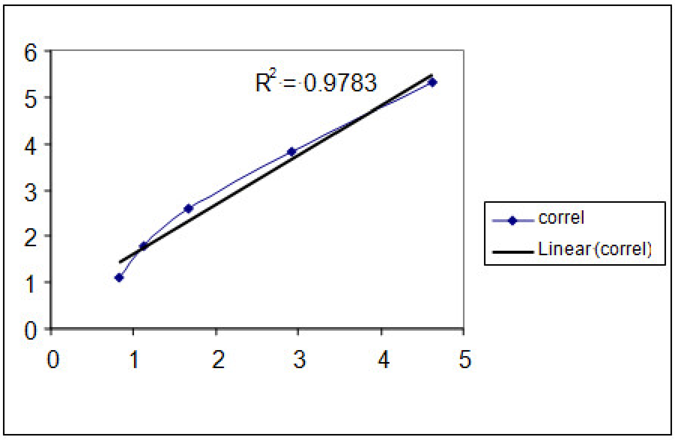

FIGURE 7. CORRELATION BETWEEN McAuliffe AND MODEL

Results and Discussion

The laboratory experimental results generated above were used to validate the model. Table 1 shows the laboratory experimental and model prediction results, viscosities and the quantity of grounded clay used. Figure 1 shows a plot of laboratory experimental result while figure 2 shows a plot both laboratory experimental and the model results. For pseudo-plastic type of emulsions (shear thinning), apparent viscosity is inversely proportional to shear rate. The lower the apparent viscosity, the higher the shear rate and vice-versa.

Similarly, the higher the flow rates, the higher the shear rates, so also the higher the pressure drops. The pressure drop in the porous medium is also dependent on the K *and* n values as well as the diameter of the material that formed the porous medium (in this case a transparent plastic cylinder).

As can be seen from table 1, with the use of grounded clay amounted between 25g and 40g, it is evident that there is a sharp decrease in pressure. This could be attributed to the quantity of the grounded clay which served as an emulsifying agent. But as more and more of this grounded clay is added, the flow of this emulsion is assuming a stabilized state, making the sharp decreased in pressure earlier witnessed to disappear. Though the pressure continued to decrease, this could be attributed to the non-Newtonian behavior of emulsion as well as the rheological properties they exhibit. It was also observed that the viscosity of pseudo-plastic type of emulsions has great effect on its shear rate as well as its pressure drop. From table 1, it can be seen that the lower the viscosity, the higher the shear rate and the higher the pressure drop in the porous medium.

The average percentage error value of 6.57% indicates the error that occurred between the predicted model results and the experimental results. This error is therefore taken as the deviation between the predicted model results and the experimental results. Therefore, using this model means that one should expect an error value of 6.57% for the result. Table 2 shows the comparison of the various models values from literature with the new model predicted.

From Figure 4 above, for Civan and the model, the correlation gives a value of $R^2 = 0.9871$. This indicates that there is a close proximity between Civan model and the model. R^2 is the correlation factor. From figure 5 above, for Marsden and the model, the correlation gives a value of $R^2 = 0.9854$. This also indicates a close proximity but a little deviation of plot when compare with that of Civan and the model. The correlation between McAuliffe and the model gives a value of $R^2 = 0.9783$. The value $R^2 = 0.9783$ shows a proximity between McAuliffe and the model but the least proximity when compare with other plots of correlation.

From Figure 6 above, for the plot of correlation between experimental and the model which gives a value of $R^2 = 0.9999$, it can be seen that this gives the best value of R^2. This also gives the best correlation plot.

Conclusion

Due to non-Newtonian attributes of emulsions and the rheological properties they exhibit, the following conclusions are established:

(i) As the viscosity of the emulsions increases, the density also increases. i.e, the viscosity of emulsions is directly proportional to density.

(ii) Flow rate is low when viscosity is high as more time is needed to displace considerable volume of emulsion in the porous medium.

(iii) The value of the viscosity of emulsions affects the starting or inlet pressure. The higher the viscosities of emulsions the higher will the inlet pressure.

(iv) It is also ascertained that the viscosity of emulsions has an indirect impact on the pressure drop in the porous medium. The lower the viscosity the higher the shear rate and the higher the pressure drop.

(v) Flow rate has direct impact on the pressure drop. The higher the flow rate the higher the pressure drop.

(vi) The average percentage error value of 6.57% occurred between the predicted model results and the experimental results.

(vii) Therefore, using this model means that one should expect an error value of 6.57% for the result. This shows that the result from the model is very reliable, simple and appropriate to determine flow of emulsions in porous media.

NOMENCLATURE

$t =$ time, secs

$\ell = density, lb / cuft$

$\varphi =$ porosity, fraction

$\Phi =$ flow potential, psi

$V =$ velocity, ft / s

$V_c =$ average fluid velocity, ft / s

$\gamma =$ shear rate, s^{-1}

$\tau =$ shear stress, $dynes / cm^2$

$Pwf =$ Flowing bottom hole pressure, psi

$q =$ Flow rate, $cuft / s$

$r =$ Radial distance, ft

$r_w =$ Wellbore radius, ft

$r_e =$ Reservoir radius, ft

$n =$ Power law index

$K =$ Consistency

$\mu =$ Viscosity, cp

$E_i =$ Exponential integral

$k =$ Permeability, md

$Dp = P_{in} =$ Inlet pressure in the porous medium, psi

$P_i =$ Pressure due to gradient, psi

$\Delta P =$ Pressure difference at various time intervals, psi

$P_L =$ Outlet pressure, psi

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APPENDIX A

Experimental Results And Calculations

Experiment 1:

$\mu = 0.10cp$

$\ell = 67lbs / cuft$

$r = 2cm = 0.066\,ft$

$h = 25cm = 0.82\,ft$

Volume displaced = 470ml = 0.47 litres

$$= \frac{0.47 * 5.615}{159.10}$$

= 0.0166 cuft

$$q = \frac{vol.\,displ.}{t}$$

$$= \frac{0.0166}{60}$$

$$0.00028 cuft / s$$

From $p_i = \ell gh$

$$= 67*32.2*0.82$$

$$= 1769.07\,psi$$

P_i is the pressure due to gradient

$$Dp = p_i - atm.p$$

$$= 1769.07 - 14.7$$

$$= 1754.37\,psi$$

Dp is the difference between the pressure due to gradient and the atmospheric pressure.This is inlet pressure in the porous medium.

From

$$P = P_i + \left[\frac{q}{2\pi h}\right]^n \frac{K}{\mu}\frac{r^{n-1}}{n+1}E_i(-\beta)$$

$$\left[\frac{q}{2\pi h}\right]^n = \left[\frac{0.00028}{2*3.142*0.82}\right]^{0.4} = 0.020$$

$$\frac{K}{\mu} = \frac{0.6}{0.10} = 6$$

$$\frac{r^{n-1}}{n+1} = \frac{0.066^{-0.6}}{1.4} = 3.65$$

$$\beta = \frac{r^{n+1}}{t}$$

At t = 300s

$$\beta = \frac{0.066^{1.4}}{300} = 0.0000742$$

$$E_i(-\beta) = E_i(-0.0000742) = \ln\left[(1.781)(0.0000742)\right]$$

$$= -8.932$$

$$P = 1754.37 + (0.020)(6)(3.65)(-8.932)$$

$$= 1754.37 + (-3.91)$$

$$= 1750.46\,psi$$

At t = 600s

$$\beta = \frac{0.066^{1.4}}{600} = 0.0000371$$

$$E_i(-0.0000371) = \ln(1.781)(0.0000371)$$

$$= -9.63$$

$$P = 1754.37 + (0.020)(6)(3.65)(-9.63)$$

$= 1754.37 + (-4.22)$

$= 1750.15\, psi$

At t = 900s

$\beta = \frac{0.066^{1.4}}{900} = 0.0000247$

$E_i(-0000247) = \ln(1.781)(0.0000247)$

-10.03

$P = 1754.37 + (0.020)(6)(3.65)(-10.03)$

$= 1754.37 + (-4.39)$

$= 1749.98\, psi$

At t 1200s

$\beta = \frac{0.066^{1.4}}{1200} = 0.0000185$

$E_i(-0.0000185) = \ln(1.781)(0.0000185)$

$= -10.32$

$P = 1754.37 + (0.020)(6)(3.65)(-10.32)$

$= 1754.37 + (-4.52)$

$= 1749.85$

At t = 1500s

$\beta = \frac{0.066^{1.4}}{1500} = 0.0000148$

$E_i(-0.0000148) = \ln(1.781)(0.0000148)$

$= -10.54$

$P_L = 1754.37 + (0.020)(6)(3.65)(-10.54)$

$= 1754.37 + (-4.62)$

$= 1749.75\, psi$

$Avg.\Delta P = \frac{1750.46 + 1750.15 + 1749.98 + 1749.85 + 1749.75}{5}$

$= \frac{8750.19}{5}$

$= 1750.04\, psi$

$P_{in} - Avg.\Delta P = 1754.37 - 1750.75$

$= 4.33\, psi$

ΔP is the pressure difference at various time intervals.

$P_{in} - P_L = 1754.37 - 1749.75$

$= 4.62\, psi$

Experiment 2:

$\mu = 0.15 cp$

$\ell = 69 lbs / cuft$

$P_i = 69 * 32.2 * 0.82$

$= 1821.9\, psia$

$\Delta P = 1821.9 - 14.7$

$= 1807\, psia$

Volume displaced = 440ml

= 0.01553cuft

$q = \frac{0.01553}{60}$

$0.00026 cuft / s$

At t = 300s

$$\left[\frac{q}{2\pi h}\right]^n = \left[\frac{0.00026}{2 * 3.142 * 0.82}\right]^{0.4} = 0.019$$

$$\frac{K}{\mu} = \frac{0.6}{0.15} = 4$$

$$\frac{r^{n-1}}{n+1} = \frac{0.066^{-0.6}}{1.4} = 3.65$$

$E_i(-\beta) = -8.932$

$P = 1807 + (0.019)(4)(3.65)(-8.932)$

$= 1807 + (-2.48)$

$= 1804.52\, psi$

At t = 600s

$E_i(-\beta) = -9.63$

$P = 1807 + (0.019)(4)(3.65)(-9.63)$

$= 1807 + (-2.68)$

$= 1804.32\, psi$

At t = 900s

$E_i(-\beta) = -10.03$

$P = 1807 + (0.019)(4)(3.65)(-10.03)$

$= 1807 + (-2.78)$

$= 1804.22\, psi$

At t = 1200s

$E_i(-\beta) = -10.32$

$P = 1807 + (0.019)(4)(3.65)(-10.32)$

$= 1807 + (-2.86)$

$= 1804.14\, psi$

At t = 1500s

$E_i(-\beta) = -10.54$

$P_L = 1807 + (0.019)(4)(3.65)(-10.54)$

$= 1807 + (-2.92)$

$= 1804.08\,psi$

$$Avg.\,Dp = \frac{1804.52 + 1804.32 + 1804.22 + 1804.14 + 1804.08}{5}$$

$= 1804.26\,psi$

$P_{in} - Avg.\Delta P = 1807 - 1804.26$

$= 2.74\,psi$

$P_{in} - P_L = 1807 - 1804.08$

$= 2.92\,psi$

Experiment 3:

$\mu = 0.25cp$

$\ell = 72 lbs / cuft$

$P_i = 72 * 32.2 * 0.82$

$= 1901\,psi$

$\Delta P = 1901 - 14.7$

$= 1886.39\,psi$

Volume displaced = 390ml

= 0.0138cuft

$$q = \frac{0.0138}{60} = 0.00023 cuft / s$$

At t = 300s

$$\left[\frac{q}{2\pi h}\right]^n = \left[\frac{0.00023}{2 * 3.142 * 0.82}\right]^n = 0.018$$

$$\frac{K}{\mu} = \frac{0.6}{0.25} = 2.4$$

$$\frac{r^{n-1}}{n+1} = 3.65$$

$E_i(-\beta) = -8.932$

$P = 1886.39 + (0.018)(2.4)(3.65)(-8.932)$

$= 1886.39 + (-1.41)$

$= 1884.98\,psi$

At t = 600s

$E_i(-\beta) = -9.63$

$P = 1886.39 + (0.018)(2.4)(3.65)(-9.63)$

$=1886.39+(-1.52)$

$=1884.87\,psi$

At t = 900s

$E_i(-\beta)=-10.03$

$P=1886.39+(0.018)(2.4)(3.65)(-10.03)$

$=1886.39+(-1.58)$

$=1884.81\,psi$

At t = 1200s

$E_i(-\beta)=-10.32$

$P=1886.39+(0.018)(2.4)(3.65)(-10.32)$

$=1886.39+(-1.627)$

$=1884.76\,psi$

At t = 1500s

$E_i(-\beta)=-10.54$

$P_L=1886.39+(0.018)(2.4)(3.65)(-10.54)$

$=1886.39+(-1.66)$

$=1884.73\,psi$

$$Avg.\Delta P=\frac{1884.98+1884.87+1884.81+1884.76+1884.73}{5}$$

$=1884.83\,psi$

$P_{in}-Avg.\Delta P=1886.39-1884.83$

$=1.56\,psi$

$P_{in}-P_L=1886.39-1884.73$

$=1.66\,psi$

Experiment 4:

$\mu=0.35cp$

$\ell=75lbs/cuft$

$P_i=75*32.2*0.82$

$=1980.3\,psi$

$\Delta P=1980.3-14.7$

$=1965.6\,psi$

Volume displaced = 330ml

= 0.0116cuft

$$q=\frac{0.0116}{60}=0.00019cuft/s$$

At t = 300s

$$\left[\frac{q}{2\pi h}\right]^n = \left[\frac{0.00019}{2*3.142*0.82}\right]^{0.4} = 0.017$$

$$\frac{K}{\mu} = \frac{0.6}{0.35} = 1.71$$

$$\frac{r^{n-1}}{n+1} = 3.65$$

$$E_i(-\beta) = -8.932$$

$$P = 1965.6 + (0.017)(1.71)(3.65)(-8.932)$$

$$= 1965.6 + (-0.95)$$

$$= 1964.65\,psi$$

At t = 600s

$$E_i(-\beta) = -9.63$$

$$P = 1965.6 + (0.017)(1.71)(3.65)(-9.63)$$

$$= 1965.6 + (-1.02)$$

$$= 1964.58\,psi$$

At t = 900s

$$E_i(-\beta) = -10.03$$

$$P = 1965.6 + (0.017)(1.71)(3.65)(-10.03)$$

$$= 1965.6 + (-1.06)$$

$$= 1964.55\,psi$$

At t = 1200s

$$E_i(-\beta) = -10.32$$

$$P = 1965.6 + (0.017)(1.71)(3.65)(-10.32)$$

$$= 1965.6 + (-1.09)$$

$$= 1964.51\,psi$$

At t = 1500s

$$E_i(-\beta) = -10.54$$

$$P_L = 1965.6 + (0.017)(1.71)(3.65)(-10.32)$$

$$= 1965.6 + (-1.12)$$

$$= 1964.48\,psi$$

$$Avg.\Delta P = \frac{1964.65 + 1964.58 + 1964.55 + 1964.51 + 1964.48}{5}$$

$$= 1964.55\,psi$$

$$P_{in} - Avg.\Delta P = 1965.6 - 1964.55$$

$$= 1.05\,psi$$

$P_{in} - P_L = 1965.6 - 1964.48$

$= 1.12\, psi$

Experiment 5:

$\mu = 0.45 cp$

$\ell = 78 lbs / cuft$

$P_i = 78 * 32.2 * 0.82$

$= 2059.5\, psi$

$\Delta P = 2059.5 - 14.7$

$= 2044.8\, psi$

Volume displaced = 280ml

= 0.0099cuft

$q = \frac{0.0099}{60} = 0.00017 cuft / s$

At t = 300s

$\left[\frac{q}{2\pi h}\right]^n = \left[\frac{0.00017}{2 * 3.142 * 0.82}\right]^{0.4} = 0.016$

$\frac{K}{\mu} = \frac{0.6}{0.45} = 1.33$

$\frac{r^{n-1}}{n+1} = 3.65$

$E_i(-\beta) = -8.932$

$P = 2044.8 + (0.016)(1.33)(3.65)(-8.932)$

$= 2044.8 + (-0.69)$

$= 2044.1\, psi$

At t = 600s

$E_i(-\beta) = 9.63$

$P = 2044.8 + (0.016)(1.33)(3.65)(-9.63)$

$= 2044.8 + (-0.75)$

$= 2044.05$

At t = 900s

$E_i(-\beta) = -10.03$

$P = 2044.8 + (0.016)(1.33)(3.65)(-10.03)$

$= 2044.8 + (-0.78)$

$= 2044.02\, psi$

At t = 1200s

$E_i(-\beta) = -10.32$

$P = 2044.8 + (0.016)(1.33)(3.65)(10.32)$

$$= 2044.8 + (-0.80)$$

$$= 2044\,psi$$

At t = 1500s

$$E_i(-\beta) = 10.54$$

$$P_L = 2044.8 + (0.016)(1.33)(3.65)(10.54)$$

$$= 2044.8 + (-0.82)$$

$$= 2043.98\,psi$$

$$Avg.\Delta P = \frac{2044.1 + 2044.05 + 2044.02 + 2044 + 2043.98}{5}$$

$$= 2044.03\,psi$$

$$P_{in} - Avg.Dp = 2044.8 - 2044.03$$

$$= 0.77\,psi$$

$$P_{in} - P_L = 2044.8 - 2043.98$$

$$= 0.82\,psi$$

APPENDIX B

Calculation of Percentage Error

From the table of results above:

Percentage error (% error) is given as;

$$\frac{Experimental - Calculated}{Calculated} * 100$$

For experiment 1:

$$\%\,error = \frac{4.62 - 4.33}{4.33} * 100$$

$$= 0.067 * 100$$

$$= 6.70$$

For experiment 2:

$$\%\,error = \frac{2.92 - 2.74}{2.74} * 100$$

$$= 0.0657 * 100$$

$$= 6.57$$

For experiment 3:

$$\%\,error = \frac{1.66 - 1.56}{1.56} * 100$$

$$= 0.0641 * 100$$

$$= 6.41$$

For experiment 4:

$$\%\,error = \frac{1.12 - 1.05}{1.05} * 100$$

$$= 0.0667 * 100$$

$$= 6.67$$

For experiment 5:

$$\% \, error = \frac{0.82 - 0.77}{0.77} * 100$$

$$= 0.0649 * 100$$

$$= 6.49$$

$$Average \, \% \, error = \frac{6.70 + 6.57 + 6.41 + 6.67 + 6.49}{5}$$

$$= \frac{32.84}{5}$$

$$= 6.568$$

$$= 6.57$$

$$The \ average \ \ percentage \ error = 6.57\%$$